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Tunnelling current across a double barrier

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Abstract : An analytical expression for tunnel current density across a double barrier has been obtained under non-resonant conditions. The derivation is based on the ideas of quantum measurement There is a good agreement with observed results in the nature of current-voltage and differential conductivity-voltage characteristics.

Keywords : Tunnel current density, double barrier, quantum measurement

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1. Introduction

Tunnelling across a double barrier was studied experimentally by Chang *et al* [1] in which they had observed resonant tunnelling under suitable conditions. Esaki and his coworkers [2–4] later applied the conventional theoretical models to explain their observations in such devices which find their application in superlattices. In recent times, Roy *et al* [5,6] have applied the ideas of quantum measurement model to study this problem.

Double barrier tunnelling continues to evince interest in workers even now. Vanhoof and his coworkers [7] have studied spatially indirect transitions due to coupling between hole accumulation layer and a quantum well in resonant tunnelling diodes. Kuznesov *et al* [8] have studied the effect of electron-electron interactions on the resonant tunnelling spectroscopy of the localised states in a barrier. Silvestrini *et al* [9] have studied resonant macroscopic quantum tunnelling in SQUID system. Alonzo and his coworkers [10] have presented a tunnelling spectroscopy of resonant interband tunnelling structures. Song [11] has presented a transition layer model and applied it to resonant tunnelling in hetero-structures.

In the present study, the quantum measurement model of Roy and his coworkers [5,6] has been used to derive an analytical expression for tunnel current density across a double barrier. Since the earlier workers had observed negative differential conductivity (n.d.c.) under non-resonant conditions in such systems, the main purpose of this work was to find a theoretical expression which could lead to n.d.c. effect.

The quantum measurement model differs from the conventional model fundamentally. The conventional model seems to rest upon the idea that the electron energy must remain unaltered throughout the tunnelling process. But tunnelling of particles is not a continually observable process. The tunnelling particle can be reckoned only after a definite time τ measured from the instant of incidence of the particle upon the barrier because of a finite time becoming necessary for potential energy estimation as required by Heisenberg's uncertainty relation. The electron is then able to recover its wave or particle shape that it once lost while making tunnelling transition. In other words, we may regard tunnelling as a process of quantum measurement being carried out by the barrier. Both energy and time being conjugate variables, simultaneous and accurate estimation of them is not possible because of Heisenberg's uncertainty relations. So, if a time τ elapses in reckoning the tunnelling process, the electron energy at the conclusion of the process must be uncertain by $\hbar/\tau$. The electron energy is expected to undergo a fluctuation of $(V_0 - E)$ energy around its original value E where V_0 represents the height of the barrier. This idea of quantum measurement has been successfully applied by Roy and other workers [12,13] to different tunnel devices.

2. Tunnelling across a double barrier

The electronic wave functions in various regions of a double barrier system (Figure 1) can be written as

$$\psi_1(x) = a_1 e^{ik_1x} + b_1 e^{-ik_1x}; \quad -\infty < x \le x_1 \tag{1}$$

$$\psi_2(x) = a_2 e^{-\chi_2(x-x_1)} + b_2 e^{\chi_2(x-x_1)}; \quad x_1 \le x \le x_2 \tag{2}$$

$$\psi_3(x) = a_3 e^{ik_3(x-x_2)} + b_3 e^{-ik_3(x-x_2)}; \quad x_2 \le x \le x_3 \tag{3}$$

$$\psi_4(x) = a_4 e^{-\chi_4(x-x_3)} + b_4 e^{\chi_4(x-x_3)}; \quad x_3 \le x \le x_4 \tag{4}$$

$$\psi_5(x) = a_5 e^{ik_5(x-x_4)}; \quad x_4 \le x \le \infty \tag{5}$$

where
$$k_1^2 = k_3^2 = k_5^2 = \frac{2m^*E}{\hbar^2} \tag{6}$$

and
$$\chi_2^2 = \chi_4^4 = \frac{2m^*(V_0 - E)}{\hbar^2}. \tag{7}$$

Here m^* is the effective mass of the electron in the double barrier system and V_0 is the barrier height.

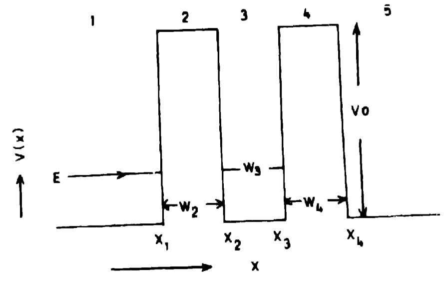

Figure 1. Double barrier system

Matching the wave functions and their first derivatives at different boundaries, one finally obtains an expression for the tunnelling probability [14,15] as

$$Z = \frac{|a_5|^2}{|a_1|^2} = \frac{2^8 k_1^2 k_5^2 \chi_2^2 \chi_4^2}{(\chi_2^2+k_1^2)(\chi_2^2+k_3^2)(\chi_4^2+k_3^2)(\chi_4^2+k_5^2)\,|K_1|^2} \tag{8}$$

where

$$\begin{aligned}
|K_1|^2 = 4\Big[& e^{2(\chi_2\omega_2+\chi_4\omega_4)} \sin^2(\phi_3+\phi_4-k_3\omega_3) \\
& + e^{2(\chi_2\omega_2-\chi_4\omega_4)} \sin^2(\phi_3-\phi_4-k_3\omega_3) \\
& + e^{-2(\chi_2\omega_2-\chi_4\omega_4)} \sin^2(\phi_3-\phi_4+k_3\omega_3) \\
& + e^{-2(\chi_2\omega_2+\chi_4\omega_4)} \sin^2(\phi_3+\phi_4+k_3\omega_3) \\
& - e^{-2\chi_2\omega_2} \cos 2\phi_5 . \sin(\phi_3+\phi_4-k_3\omega_3)\sin(\phi_3-\phi_4-k_3\omega_3) \\
& + 2e^{2\chi_4\omega_4} \cos 2\phi_2 . \sin(\phi_3+\phi_4-k_3\omega_3)\sin(\phi_3-\phi_4+k_3\omega_3) \\
& - 2\cos\{2(\phi_2+\phi_5)\}.\sin(\phi_3+\phi_4-k_3\omega_3)\sin(\phi_3+\phi_4+k_3\omega_3) \\
& - 2\cos\{2(\phi_2-\phi_5)\}.\sin(\phi_3-\phi_4-k_3\omega_3)\sin(\phi_3-\phi_4+k_3\omega_3) \\
& + 2e^{-\chi_4\omega_4} \cos 2\phi_2 . \sin(\phi_3-\phi_4-k_3\omega_3)\sin(\phi_3+\phi_4+k_3\omega_3) \\
& - 2e^{2\chi_2\omega_2} \cos 2\phi_5 . \sin(\phi_3-\phi_4+k_3\omega_3)\sin(\phi_3+\phi_4+k_3\omega_3)\Big],
\end{aligned} \tag{9}$$

where

$$\phi_2 = \tan^{-1}\left(\frac{\chi_2}{k_1}\right), \qquad \phi_3 = \tan^{-1}\left(\frac{\chi_2}{k_3}\right),$$

$$\phi_4 = \tan^{-1}\left(\frac{\chi_4}{k_3}\right) \quad \text{and} \quad \phi_5 = \tan^{-1}\left(\frac{\chi_4}{k_5}\right). \tag{10}$$

It is found that a resonance is obtained *i.e.* $Z = 1$ when the following conditions are satisfied simultaneously

(i) $k_1 = \chi_2 = k_3 = \chi_4 = k_5$,

which leads to $\phi_2 = \phi_3 = \phi_4 = \phi_5 = \frac{\pi}{4}$,

(ii) $\chi_2 \omega_2 = \chi_4 \omega_4$

and (iii) $k_3 \omega_3 = \left(n + \frac{1}{2}\right)\pi$ where $n = 0, 1, 2 \ldots.$

3. Tunnel current density

Regions 2 and 4 of Figure 1 are barriers whereas region 3 is a potential well. The current in this system flows by the quantum measurement mechanism through regions 2 and 4 and by conventional mechanism through region 3. The principle of continuity suggests that the current densities through all these regions must be the same.

The one-electron tunnel current density through region 4, generated by quantum measurement process, is given by [13]

$$J_4 = J_{01} \frac{\sin \omega_{lr} \tau}{\omega_{lr} \tau} + J_{02} \sin\left(\omega_{lr} \tau + \theta\right), \tag{11}$$

where

$$J_{01} = \frac{q.2\hbar \chi_4}{m^*} \mid a_4 \mid^2 e^{-2\chi_4 \omega_4}, \tag{12}$$

which on further simplification leads to

$$J_{01} = \frac{q\hbar}{2m^* \chi_4} \left(k_5^2 + \chi_4^2\right) Z \mid a_1 \mid^2, \tag{13}$$

$$\omega_{lr} = \frac{E_l - E_r}{\hbar}, \quad \tau = \frac{2m^*}{h\chi_4^2} \text{ the tunnelling time.}$$

When a group of electrons having a random phase difference amongst themselves is incident upon the double barrier system, the differential tunnel current density is given by [12,13]

$$dJ(E) = \rho_l(E) f_l(E) dE \sum_{-\infty}^{\infty} \left[J_{01} \frac{\sin \omega_{lr} \tau}{\omega_{lr} \tau} + J_{02} \sin\left(\omega_{lr} \tau + \theta\right)\right], \tag{14}$$

where $\rho_l(E) f_l(E)\, dE$ is the density of the wave group at the incident end.

The minimum phase difference at the transmitted end is

$$d\left(\omega_{lr} \tau\right) = \frac{\epsilon_l \tau}{\hbar}, \tag{15}$$

where ϵ_r is the difference in consecutive energy levels at that end. Thus, the summation of (14) can be converted into integration as

$$dJ(E) = \rho_l(E)f_l(E)dE\left(\frac{\pi\hbar}{\epsilon_r\,\tau}\right)\left[J_{01}\int_{-\infty}^{\infty}\frac{\sin\omega_{lr}\tau}{\omega_{lr}\tau}d(\omega_{lr}\tau)\right.$$

$$\left.+J_{02}\int_{-\infty}^{\infty}\sin(\omega_{lr}\tau+\theta)d(\omega_{lr}\tau)\right]$$

which finally leads to [12,13]

$$dJ(E) = \left(\frac{\pi\hbar}{\epsilon_r\,\tau}\right)J_{01}\,\rho_l(E)f_l(E)dE. \tag{16}$$

But ϵ_r can be expressed in terms of the density of states ρ_r as

$$\frac{1}{\epsilon_r} = \rho_r(E)\left[1-f_r(E)\right]\Omega, \tag{17}$$

where Ω is the volume of the electrode at the transmitted end. Substituting (17) in (16), we get

$$dJ(E) = \frac{\pi\hbar\,\Omega}{\tau}J_{01}f_l(E)\left[1-f_r(E)\right]\rho_l(E)\rho_r(E)dE. \tag{18}$$

For absolute zero temperature, $f_l(E)\left[1-f_r(E)\right] = 1$ and hence

$$dJ(E) = \frac{\pi\hbar\,\Omega}{\tau}J_{01}\,\rho_l(E)\rho_r(E)dE. \tag{19}$$

Substituting for J_{01} from (13) and putting

$$\tau = \frac{2m^*}{\hbar\chi_4^2} = \frac{\hbar}{V_0-E}, \text{ one finally obtains}$$

$$dJ(E) = \frac{2^{29/2}\,\pi\Omega q m^{*5/2}\,|a_1|^2\,E^3(V_0-E)^{5/2}\,dE}{\hbar^6\,V_0^3\,|K_1|^2}. \tag{20}$$

After certain simplifications, it is found that

$$|K_1|^2 = 4\sin^2\left(2\phi_2-k_3\omega_3\right)e^{4\chi_2\omega_2}, \tag{21}$$

where $\chi_2\omega_2 = \chi_4\omega_4$

Thus, one gets

$$dJ(E) = P\,\frac{E^3(V_0-E)^{5/2}\,e^{-K(V_0-E)^{1/2}}}{\sin^2\left(2\phi_2-K_3\omega_3\right)}dE, \tag{22}$$

$$\left.\begin{aligned} \text{where} \quad P &= \frac{2^{19/2}\,\pi\Omega q m^{*5/2}\,|\,a_1\,|^2}{\hbar^6 V_0^3} \\ \text{and} \quad K &= \frac{4\sqrt{2m}^{*1/2}\,\omega_2}{\hbar}. \end{aligned}\right\} \tag{23}$$

The tunnel current density can be expressed as

$$J(E) = P\int_0^{qV} \frac{E^3(V_0 - E)^{5/2}\, e^{-K(V_0-E)^{1/2}}\, dE}{\sin^2\left(2\phi_2 - k_3\omega_3\right)}, \tag{24}$$

where V is the applied bias. Eq. (24) has been obtained for non-resonant conditions.

4. Results and conclusions

The integration was done numerically with the help of a computer. The current-voltage characteristics for AlGaAS–GaAS–AlGaAS as double barrier system, are shown in

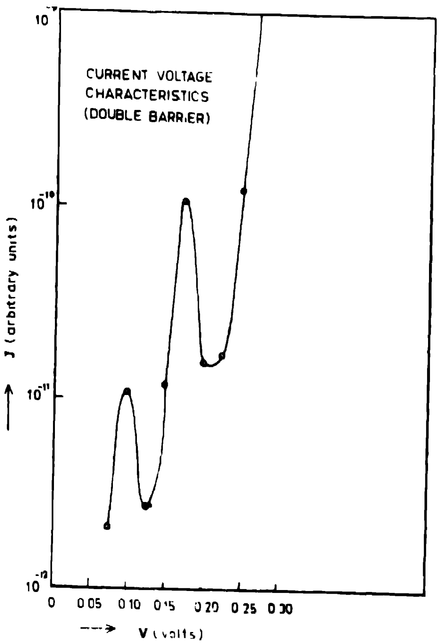

Figure 2. Current-voltage characteristics

Figures 2 and 3 for different ranges of biases. Negative resistance regions are clearly seen in these plots. Figure 4 shows the differential conductivity-voltage characteristics. One finds a good agreement so far as the nature of these characteristics are concerned.

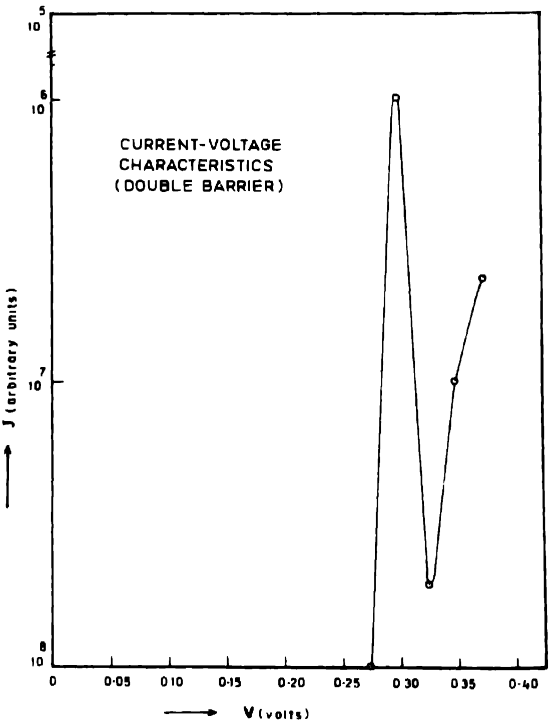

Figure 3. Current-voltage characteristics.

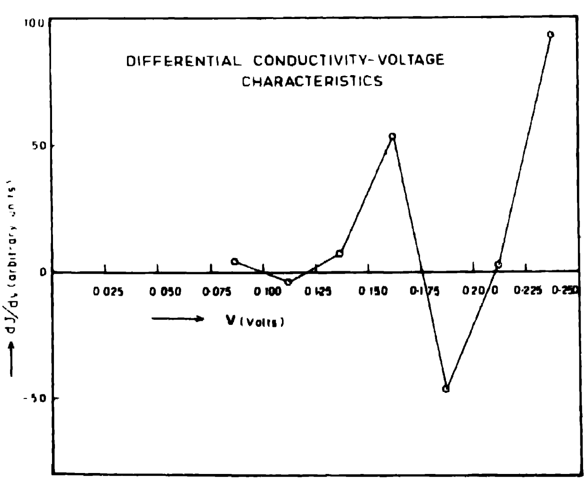

Figure 4. Differential conductivity-voltage characteristics

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